Geometrical methods in Dynamics and Topology

Hanoi, April 18-22, 2011

Basic information:

This Conference will celebrate the 60th anniversary of the Hanoi National University of Education. The main topics for the conference are Geometry, Topology and Dynamical systems with a special emphasis on their symplectic aspects..

The conference will start on Monday 18th and will finish on Friday 22nd. There is an excursion to Halong-Bay envisaged for the weekend 23-24. We also plan to have Wednesday's afternoon free.

Latest updates

Webpage created, September 2010

Information about Visa added in December 6, 2010 (go to Practical Infomation above)

Venue

The conference will take place at Hanoi National University of Education.

Lake Hoan Kiem

Our poster:

Download our poster here.

Contact

For any queries concerning scientifical or practical aspects of the conference please contact any of the organizers. For comments concerning this webpage, contact Eva Miranda. If you want to register for this conference please send the following information to Eva Miranda (this conference is partially supported by the ESF and this information will be included in a list for the ESF):

Name:

Gender:

Nationality:

Affiliation:

Affiliation's address:

Geometrical methods in Dynamics and Topology

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Venue Speakers Funding Organizers Programme Practical Information Pictures
Proceedings

There will be minicourses by:

Alain Chenciner (Observatoire de Paris-IMCCE): ***"The Lagrange reduction of the N-body problem"***

*Extra conference by Alain Chenciner on: "The angular momentum of a relative equilibrium
in dimensions higher than 3" abstract*

Nguyen Tien Zung (Toulouse) "Entropy in Physics and Mathematics"

List of speakers:

Marta Batoreo (UC Santa Cruz)

Marc Chaperon (Paris VII)

Alain Chenciner (IMCEE)

Jesus Gonzalo (Madrid)

Basak Gurel (Vanderbilt)

Mark Hamilton (Mount Allison)

Janko Latschev (Hamburg University)

Do Ngoc Diep (Hanoi)

Ricardo Perez Marco (Paris XIII)

Miguel Rodriguez-Olmos (Universitat Politecnica de Catalunya)

Dietmar Salamon (ETH-Zurich)

Romero Solha (Univesitat Politecnica de Catalunya)

Sheila Sandon (Nantes)

Michael Usher (University of Georgia)

Le Anh Vu (Hanoi)

Le Van Hong (Praha)

Vu The Khoi (Hanoi) Dmitri Zaitsev (Trinity College)	Skinny houses in Hanoi

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The 3 main local sponsors for our conference are:

- Hanoi National University of Education
- Vietnam National Science Foundation (NAFOSTED)–
- Hanoi Institute of Mathematics

The conference is also partially supported by:

- The European Science Foundation via CAST (Contact and Symplectic Topology)

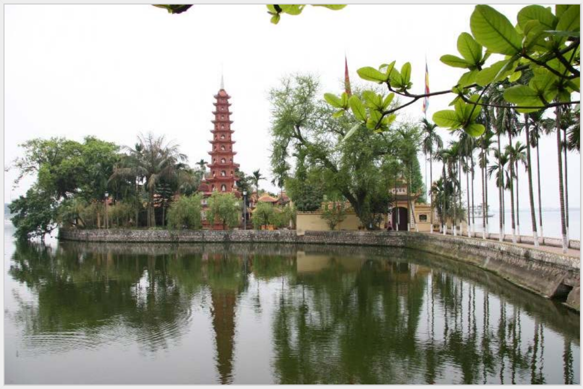
HANOI NATIONAL UNIVERSITY OF EDUCATION

NAFOSTED Quỹ phát triển khoa học & công nghệ quốc gia

Vietnam Academy of Science and Technology INSTITUTE of MATHEMATICS

EUROPEAN SCIENCE FOUNDATION SETTING SCIENCE AGENDAS FOR EUROPE

The conference will take place at Hanoi National University of Education



Tran Quoc Pagoda

Geometrical methods in Dynamics and Topology

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Organizers:

Viktor Ginzburg

Eva Miranda

Do Duc Thai

Nguyen Tien Zung

Kaleidoscopic vision of Skinny houses in Hanoi

Geometrical methods in Dynamics and Topology

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Programme:

Schedule:

Time	Monday	Tuesday	Wednesday	Thursday	Friday	Weekend-HalongBay
9:30-11	(Opening) Chenciner	Zung	Chenciner	Zung	Chenciner (1-hour talk)	EXCURSION TO HALONG BAY
Break						
11:30-12:30	Salamon	Gonzalo	Perez-Marco	Gurel	Zaitsev	
LUNCH						
14:00-15:00	Vu The Khoi	Latschev	FREE AFTERNOON	Hamilton	Rodriguez-Olmos	
15:00-16:00	Usher	Le Anh Vu		Le Van Hong	Solha	
Break						
16:30-17:30	Do Ngo Diep	Sandon		Batoreo	(Closing) Chaperon	

Colour's code: In orange: Minicourses (1 hour 30 minutes)

In Turquoise-blue: 1-hour talks

Note: There will be a Banquet of Thursday's night.

On Friday talks will start at 10.00 am.

Minicourses:

Alain Chenciner (Observatoire de Paris-IMCCE): *"The Lagrange reduction of the N-body problem"*

Abstract :
In his fondamental "Essai sur le problème des trois corps" (Essay on the 3-body problem), Lagrange, well before Jacobi's "reduction of the node", carries out the first complete reduction of symetries in this problem. Discovering the so-called homographic motions (Euler had treated only the colinear case), he shows that these motions necessarily take place in a fixed plane, a result which is simple only for the "relative equilibria". In order to understand the true nature of this reduction -- and of Lagrange's equations - it is necessary to consider the n-body problem in an euclidean space of arbitrary dimension. The actual dimension of the ambiant space then appears as a constraint, namely the the angular momentum bivector's degeneracy. I shall describe the results obtained in a joint paper with Alain Albouy published (in french !) in 1998. For a non homothetic homographic motion to exist, it is necessary that the motion takes place in an even dimensional space E. Two cases are possible: either the configuration is ``central" (that is a critical point of the potential among configurations with a given moment of inertia) and the space E is endowed with an hermitian structure, or it is ``balanced" (that is a critical point of the potential among configuration with a given inertia spectrum) and the motion is a new type, quasi-periodic, of relative equilibrium.

References:

J.L. Lagrange, Essai sur le problème des trois corps, Oeuvres volume 6, pages 229-324 (1772)

E. Betti, Sopra il moto di un sistema di un numero qualunque di punti che si attragono o si respingono tra di loro, Annali di Matematica s.2 t. 8, pages 301-311 (1877)

A. Wintner, The analytical foundations of Celestial Mechanics, Princeton University Press (1941)

A. Albouy & A. Chenciner, Le problème des n corps et les distances mutuelles, Inventiones mathematicae 131, pages 151-184 (1998)

A. Albouy, Mutual distances in Celestial Mechanics, Lectures at Nankai Institute, Tianjin, Chine, juin 2004

Nguyen Tien Zung (Toulouse) *"Entropy in Physics and Mathematics"*

Abstract :
In this minicrouse, intended to be accessible also to undergraduate students, I'll try to explain various manifestiations of the concept of entropy, including Shannon entropy in information theory (amount of information), Clausius-Boltzmann entropy in physics (laws of thermodynamics), Kolmogorov-Sinai entropy in dynamical systems (rate of information), entropy in geometry problems (e.g. isoperimetric inequalities), etc. In the last part of this minicourse I'll talk about my little results on the entropy of geometric

structures, which generalizes the notion of geometric entropy of folations introduced by Ghys--Langevin--Walczak.

Lecturers:

Marta Batoreo (UC Santa Cruz):*"On the Rigidity of the Maslov Index for Coisotropic Submanifolds of Rational Symplectic Manifolds"*

Abstract: A result of V. Ginzburg on the rigidity of the coisotropic Maslov index asserts that there exists a non-trivial loop (tangent to the characteristic foliation of a stable coisotropic submanifold of a symplectically aspherical manifold) with certain bounds on its Maslov index and area. This theorem also holds for some symplectic manifolds not necessarily symplectically aspherical. We shall state the theorem for the rational case and sketch its proof.

Marc Chaperon (Paris VII)***" Generalized Hopf bifurcations"***

Abstract:Following Thom's principle: "Always look for the organizing center of phenomena", I was able to prove a general "birth lemma" for families of dynamical systems at partially elliptic rest points; as noticed by Santiago López de Medrano, it implies the birth of normally hyperbolic compact invariant manifolds diffeomorphic to all kinds of moment-angle manifolds in generic families; these manifolds can form a family of matrioshkas, providing for example a very simple model for the transition between two periodic regime

Alain Chenciner (Observatoire de Paris-IMCCE): *"The angular momentum of a relative equilibrium in dimensions higher than 3"* abstract

Jesus Gonzalo (Madrid), ***"Flow dynamics and the existence of contact circles"***

Abstract:For flows tangent to plane fields in dimension 3 we define a twisting property, related to the Anosov property. We shall explain how this property gives a simple geometric understanding
of contact circles, which are pairs of contact structures discovered and studied by H. Geiges and the speaker. We shall mention open questions and challenges provided by this connection. For example one can use Seiberg-Witten to exhibit plane fields none of whose tangent flows satisfy the twisting property.

Basak Gurel (Vanderbilt) *"Conley conjecture for negative monotone symplectic manifolds"*

Abstract: The Conley conjecture, formulated by Conley in 1984, asserts the existence of infinitely many periodic orbits for Hamiltonian diffeomorphisms of tori and was established by Hingston in 2004. Of course, one can expect the conjecture to hold for a much broader class of closed manifolds and this is indeed the case. For instance, by now, it has been proved for all closed, symplectically aspherical manifolds and Calabi-Yau manifolds using symplectic topological methods. Most recently, jointly with Ginzburg, we establish the conjecture for negative monotone, closed symplectic manifolds.

In this talk, based on joint works with Ginzburg, we will examine the question of existence of infinitely many periodic orbits for Hamiltonian diffeomorphisms and outline a proof of the Conley conjecture in the negative monotone case.

Mark Hamilton (Mount Allison), *"Real and Kahler quantization of flag manifolds."*

Janko Latschev (Hamburg University) ***"Algebraic torsion in contact manifolds***

AbstractIn this talk, I will report on recent joint work with Chris Wendl which uses the algebraic formalism of symplectic field theory to define a hierarchy of obstructions to symplectic filling of and exact symplectic corbordisms between given contact manifolds.

I will also present 3-dimensional examples to show that these obstructions are non-trivial and distinct.

Do Ngoc Diep (Institute of Mathematics, VAST) ***"Geometric Quantization of Fields Based on Geometric Langlands Correspondence"***

Abstract:We expose a new procedure of quantization of fields, based on the Geometric Langlands Correspondence. Starting from fields in the target space, we first reduce them to the case of fields on one-complex-variable target space, at the same time increasing the possible symmetry group G^L. Use the sigma model and momentum maps, we reduce the problem to a problem of quantization of trivial vector bundles with connection over the space dual to the Lie algebra of the symmetry group G^L. After that we quantize the vector bundles with connection over the coadjoint orbits of the symmetry group G^L. Use the electric-magnetic duality to pass to the Langlands dual Lie group G. Therefore, we have some affine Kac-Moody loop algebra of meromorphic functions with values in Lie algebra Lie(G). Use the construction of Fock space reprsentations to have representations of such affine loop algebra. And finally, we have the automorphic representations of the corresponding Langlands-dual Lie groups G. http://www.hindawi.com/journals/ijmms/2009/749631/:

Ricardo Perez Marco (Paris XIII)" *Total integrability or general non-integrability in Hamiltonian Dynamics"*

Miguel Rodríguez-Olmos (Universitat Politecnica de Catalunya) "***Symmetric Hamiltonian bifurcations and isotropy"***

Abstract:We will use the bundle equations of Roberts et al for symmetric Hamiltonian systems in order to study some aspects of the stability and bifurcations of relative equilibria. It will be shown how the existence of continuous isotropy groups for the symmetry action can affect these properties with respect to the free case. Notably, we will discuss how these isotropy groups can induce bifurcations from a formally stable branch of relative equilibria. This is a joint work with J. Montaldi.

Dietmar Salamon (ETH-Zurich): ***"Uniqueness of symplectic structures."***

Sheila Sandon (Nantes) ***"On existence of translated points for contactomorphisms"***

Abstract:A point p in a contact manifold is called a translated point for a contactomorphism \phi with respect to a fixed contact form if p and \phi(p) belong to the same Reeb orbit and if the contact form is preserved at p. In my talk I will discuss the problem of existence of translated points, and its relation with the Arnold conjecture, the chord conjecture and the problem of leafwise coisotropic intersections. If I will have the time I will also explain how to use generating functions techniques to study this problem for contactomorphisms of the euclidean space, the sphere and the projective space.

Romero Solha (Univesitat Politecnica de Catalunya) ***"Foliated cohomology and geometric quantisation of integrable systems with singularities"***

Abstract: This talk shows an attempt to extend some results by Snyaticki, Guillemin and Sternberg in geometric quantisation considering regular fibrations as real polarisations to the singular setting. The real polarisations concerned here are given by integrable systems with nondegenerate singularities (in the Morse-Bott sense). And the definition of geometric quantisation used is the one suggested by Kostant; via higher cohomology groups. The case of nondegenerate singularities was obtained in dimension 2 by Hamilton and Miranda and the completely elliptic case was considered by Hamilton in any dimension. The approach is to combine previous results of Miranda and Presas on a Künneth formula to reduce to the 2-dimensional case with an extension of the results of Rawnsley on the Kostant complex. This talk is based on joint work in progress with Eva Miranda.

Michael Usher (University of Georgia) ***"Aperiodic symplectic manifolds"***

Abstract: We describe a general construction which, on a very diversefamily of closed manifolds, gives rise to symplectic forms that admit autonomous Hamiltonian flows with no nontrivial periodic orbits. In particular, our family includes many of the classic examples of interesting symplectic four-manifolds with b+ > 1. This contrasts with a result of Lu which, when combined with results from Taubes-Seiberg-Witten theory, shows that such symplectic forms can never exist on manifolds with b+ = 1. All this suggests a number of open questions, some of which we will discuss.

Le Anh Vu (Hanoi): ***" The Structure of Coadjoint Orbits (K-orbits) of a Class of Real Solvable Lie Groups"***

Abstract: The talk introduces the Kirillov’s bilinear form and the symplectic structure on each K-orbit of an Lie group. We also introduce an overview of foliations formed by the family of maximal dimensional K-orbits (MD- foliations) of solvable real, simply connected Lie groups such that its K-orbits are either orbits of dimension zero or orbits with maximal dimensions (MD-groups). In addition, the talk gives analytical description or characterization Connes' C*-algebras of these foliations by KK-functors.

Le Van Hong (Praha) *"Smooth structures on stratified symplectic spaces"**slides here*

Abstract. We introduce the notion of a smooth structure on a stratified symplectic space. We show that under a mild condition many properties of a symplectic manifold can be extended to a symplectic stratified space provided with a smooth Poisson structure, e.g. the existence and uniqueness of a Hamiltonian flow, the isomorphism between the Brylinski-Poisson homology and the de Rham homology, the existence of a Leftschetz decomposition on a symplectic stratified space.:

Vu The Khoi, *"ON THE BURNS-EPSTEIN INVARIANTS OF SPHERICAL CR 3-MANIFOLDS"*

Abstract.Abstract: In this talk we present a method to compute the Burns-Epstein invariant of a spherical CR homology sphere, up to an in-teger, from its holonomy representation.

Dmitri Zaitsev (Trinity College) *"Dynamics of one-resonant biholomorphisms"*

Abstract:We construct a simple formal normal form for holomorphic diffeomorphisms in C^n whose differentials have one-dimensional family of resonances in the first m eigenvalues, $m\leq n$ (but more resonances are allowed for other eigenvalues).Next, we provide invariants and give conditions for the existence of basins of attraction. Finally, we give applications and examples demonstrating the sharpness of our conditions. This is a joint work with Filippo Bracci.

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This page includes information about

- VISA
- HOTEL
- SOCIAL PROGRAMME
- PLANE TICKETS

VISA:.

You'll need a visa to enter Vietnam. Since you will attend a conference in Vietnam, the Vietnamese side will issue a kind of "official" visa for you. (Of course, you may also "do it yourself" and ask for a tourist visa at a Vietnamese Consulate if you wish).

Do Duc Thai (Prof. at Hanoi National University of Education) will take care of this visa problem for the conference. He will send a request to the Interior Ministry of Vietnam, with the list of foreign participants of the conference. The Interior Ministry will then issue an approval letter for each participant, send that letter by fax to the corresponding Consulate (where you want to pick up your visa), asking the Consulate to issue a visa for you. Each letter has a reference number. We will inform you of the reference number. Then you can go to the

PLANE TICKETS: Due to an significant increase of the number of tourists in Vietnam, it may be very difficult to reserve plane tickets for the dates that you wish at reasonable prices. So please think about reserving your tickets as soon as possible !
Our latest information is that the prices have increased from last year, and it now costs about 1200 Euros for a round trip Paris-Hanoi ou Saigon (taxes included) with Vietnam Airlines, and a bit higher with Air France. There are other options, e.g. with Singapour Airlines, Thai Airlines, Lufthansa, Japan Airlines, American Airlines (for the US), etc.
Hint: Don't trust online travel agencies for tickets to Vietnam, because those online portals don't show "negotiated prices" and show only more expensive prices in general. Instead, call a "brick and mortar" agency and ask for best prices.

In France, you may try these 2 agencies which are specialized in trips to Asia:

* Hit Voyages, 21 Rue des Bernardins - 75005 PARIS

Tel. : 01 43 54 17 17 - Fax : 01 43 25 22 16
* Ariane Tours, 5 square Dunois 75013 PARIS
Tel: 01 45 86 88 66 - Fax: 01 45 82 21 54 .

Consulate or send your passport there, together with the reference number, and they'll issue the visa for you.

Please send to Do Duc Thai (ducthai.do@gmail.com) as soon as possible (**we would really appreciate if you could do it before February 10th!**) the following information:

1. Your surname and given names
3. Date and place of birth
4. Nationality
5. Professional address
6. Home address
7. Passport: number, date of issue, date of expiry, the issuing authority
8. At which Vietnamese Embassy will you pick up your visa ?

(all the above fields are mandatory)

9. Scanned file of main pages of your passport

If you have any accompanying person, then we'll need the same information concerning him/her too.

(Please note that the Vietnamese Interior Ministry takes at least 1 month in order to issue the letter, so we need to send them the request before February 15th in order to ensure that everything goes smoothly)

If you have any question/problem, please don't hesitate to contact Do Duc Thai for this.
Thank you very much!

Some addresses:
EMBASSY OF VIETNAM IN PARIS, FRANCE
Address: 62-66 Rue Boileau, 75016 Paris, FRANCE

Telephone: (331) 4414 6400 Fax: (331) 4524 3948

Embassy of Vietnam in Madrid (the only one in Spain)EMBAJADA DE VIETNAM EN ESPANA
C/ Segre 5
28002 MADRID - ESPANA
Tel : 0034 91 5102867 Fax : 0034 91 4157067

P/S: If you need an official invitation letter for Hanoi Geometry Conference, please just let us know, and the Hanoi National University of Education (the organizing institution for the conference) will send you one promptly.

HOTEL

By default, we will reserve rooms at the following hotel for most
participants: Army Hotel, 33 C Pham Ngu Lao Street, Hanoi, Vietnam
http://www.vietnamstay.com/hotel/army/
http://www.tripadvisor.com/Hotel_Review-g293924-d454987-Reviews-Army_Hotel-Hanoi.html

This hotel is situated in a quiet street in the center of Hanoi (walking distance from many points of interest). It has a swimming pool (free access). The negotiated price is about $60/day/single room including breakfast.

If you prefer to reserve a hotel by yourself, please send an email to Do Duc Thai to let him know that, so that or local organizers will not reserve a hotel for you !

If you need a double room or a suite (for a family), or want to share a double room with another participant, please also let us know!

Since the University is not in the center of Hanoi, we will request a special university bus to take the participants from the hotel to the university and back every conference day.

If you prefer to look for a hotel by yourself, here are some possible places to start:

- http://www.hanoihotel.net/home/
- http://www.tripadvisor.co.uk/Hotels-g293924-Hanoi-Hotels.html
- http://www.vietnamstay.com/hotel/hanoi.htm

(Prices vary from under $20 to above $100 per night)

SOCIAL PROGRAM includes:

- A conference banquet (on Thursday).
- Excursion of Hanoi: Wednesday 20/Apr afternoon 14h-19h (visit of the Temple of Literature, "But Thap" Pagoda, etc.).
- 2-day excursion to Halong Bay:

Excursion to HaLong Bay

Halong Bay is a marvellous place in the World. The Halong Bay is also an UNESCO heritage.

We hope the participants of our Conference will enjoy this excursion.

Here is the detailed plan of our excursion:

1. Saturday 23 rd:

+) We will leave the Army Hotel for Halong Bay at 9am.

+) Lunch in HaiDuong around 11:30am.

+) From 2pm to 4pm: Visit KIEP BAC and CON SON temples.

+) Arrive at the Halong Pearl Hotel around 6pm.

(This is a four stars Hotel, see Website: www.halongpearl.vn)

+ Dinner: at the same Hotel.

+ Evening: Free for visiting Halong City.

2. Sunday 24th:

+) Breakfast: 7am at the same Hotel. After that check-out.

+) 7:30am leaving the Hotel for visiting to the Halong Bay.

+) Lunch on the boat

+) We will leave Halong Bay for the Noi Bai International Airport, after that come back to the Army Hotel (We will arrive at the Noi Bai Airport around 4pm).

Total price: 100USD + 10 USD (for tax) = 110 USD

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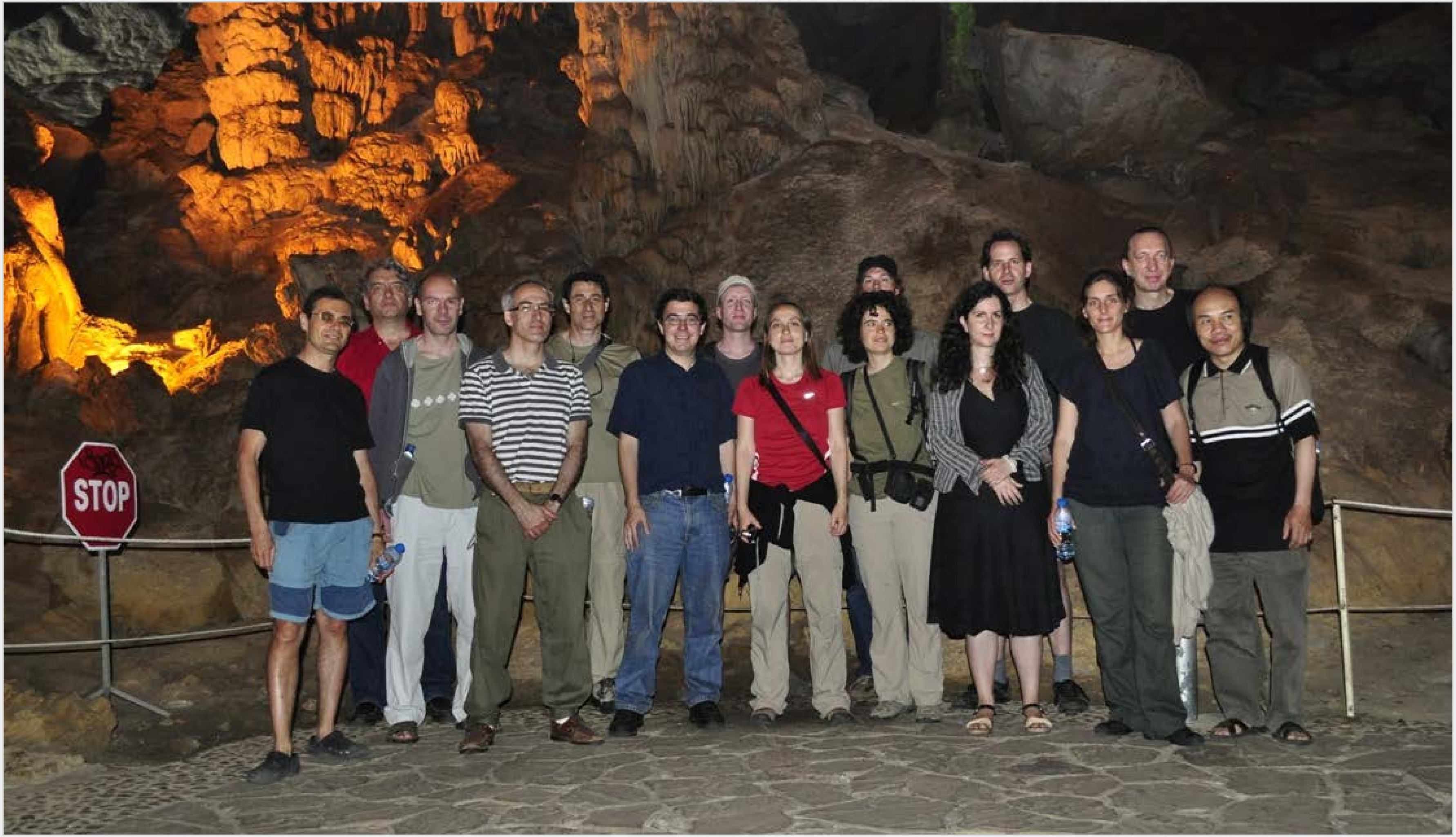
In this link you will find some pictures of the conference:

http://www-ma1.upc.es/~miranda/hanoi/conference/pictures/index.html


STOP

Geometrical methods in Dynamics and Topology

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The proceedings of the conference will be published at Acta Mathematica Vietnamica:

http://www.math.ac.vn/publications/acta/

You can either submit an original article or a survey article about the subject.

Deadline: 31, December 2011.

April 18th-22nd, 2011

GEDYTO: Geometrical Methods in Dynamics and Topology

Hanoi National University of Education, Hanoi (Vietnam)

Invited lectures

Marta Batoreo (UC Santa Cruz)

Marc Chaperon (Paris VII)

Alain Chenciner (IMCEE)

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Le Van Hong (Praha)

Alberto Verjovsky (Cuernavaca)

Dmitri Zaitsev (Trinity College)

Minicourses

Alain Chenciner (IMCEE)

Nguyen Tien Zung (Toulouse)

Organizers

Viktor Ginzburg (Santa Cruz)

Eva Miranda (Barcelona)

Do Duc Thai (Hanoi)

Nguyen Tien Zung (Toulouse)

More information about the event can be found online: http://www-ma1.upc.es/~miranda/hanoi/conference/

Vietnam Academy of Science and Technology INSTITUTE of MATHEMATICS

The angular momentum of a relative equilibrium in dimensions higher than 3

A. Chenciner

Abstract. In an euclidean space of (necessarily) even dimension $2p$, the relative equilibrium motions of a central (or balanced) n-body configuration are in one-to-one correspondence with complex structures. As soon as p is greater than 1, there are many of them. The angular momentum of such a motion may be identified with a $2p \times 2p$ anntisymmetric matrix and its spectrum is of the form $\pm i\nu_1, \cdots, \pm i\nu_p$. One asks for the determination of the subset of $\mathbb{R}^p$ formed by the ordered p-tuples $\nu_1 \geq \nu_2 \geq \cdots \geq \nu_p$ which are associated in this way to a given central configuration. The answer we obtain for $p = 2$ leads to a conjecture which links this problem to the so-called *Horn problem* which studies the spectrum of a sum of two hermitian (or real symmetric) matrices whose spectra are given.

Ref. `http://arxiv.org/abs/1102.0025`

Smooth structures on stratified symplectic spaces

Hông Vân Lê

Mathematical Institute of ASCR, Praha

GEDYTO, Hanoi, April 18-22, 2011

Outline

1. Stratified spaces and their smooth structures
   - Stratified spaces
   - Smooth structures on stratified spaces
   - Structures theorems
2. Compatible smooth structures on symplectic stratified spaces
   - Symplectic stratified spaces
   - Compatible smooth structures
3. Properties of compatible smooth structures on stratified symplectic spaces
   - Brylinski-Poisson homology
   - Existence of Hamiltonian flow
   - Leftschetz decomposition
4. End

Outline

Decomposed spaces

- Let $(X, \mathcal{S})$ be a Hausdorff and paracompact topological space of finite dimension with ordered relation.
- A $\mathcal{S}$-decomposition of X is

$$X = \cup_{i \in \mathcal{S}} S_i ,$$

where X_i are locally closed smooth manifolds
- such that

$$S_i \cap \bar{S}_j \neq \emptyset \Longleftrightarrow S_i \subset \bar{S}_j \Longleftrightarrow i \leq j .$$

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Stratified spaces

A cone $cL := L \times [0, \infty]/L \times \{0\}$.

A decomposed space X is called a stratified space if for any x in a stratum S there exists an open neighborhood $U(x) \subset X$, an open ball $B(x) \subset S$, a compact stratified space L, called the link of x, and a stratified diffeomorphism $\phi : B \times cL \to U$ that **preserves the decomposition.**

$depth_X S := \sup\{n|$ there exist pieces $S = S^0 < S^1 < \cdots < S^n\}$.

$$depth\, X := \sup_{i \in \mathcal{S}} depth\, S_i.$$

X^{reg} is assumed to be connected.

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A decomposed space X is called a stratified space if for any x in a stratum S there exists an open neighborhood $U(x) \subset X$, an open ball $B(x) \subset S$, a compact stratified space L, called the link of x, and a stratified diffeomorphism $\phi : B \times cL \to U$ that **preserves the decomposition.**

$depth_X\, S := \sup\{n|$ there exist pieces $S = S^0 < S^1 < \cdots < S^n\}$.

$$depth\, X := \sup_{i \in \mathcal{S}} depth\, S_i .$$

X^{reg} is assumed to be connected.

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Examples of pseudomanifolds w.i.c.s.

- Let M be a smooth manifold with boundary $\partial M = L = \cup L_i$.
  $M_1 := M \cup cL := L \times [0,1]/L \times \{0\}$,
  $M_2 : M \cup_i cL_i$.
- The quadric $Q_m = \{z \in \mathbb{C}^{m+1} | \sum_{i=1}^{m+1} z_i^2 = 0\}$ with isolated singularity at 0 is a pseudomanifold w.i.c.s..
  $Q_m = cL$, where $L = V_{2,m+1} = SO(m+1)/SO(m-1)$.
- Any smooth manifold with k marked points is a pseudomanifold w.i.c.s.

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Smooth structures on a pseudomanifold w.i.c.s.

A smooth structure on a pseudomanifold w.i.c.s. M is a choice of a $\mathbb{R}$-subalgebra $C^\infty(M) \subset C^0(M)$ satisfying the following three properties.

1. $C^\infty(M)$ is a germ-defined C^∞-ring, i.e. it is the C^∞-ring of all sections of a sheaf $SC^\infty(M)$ of continuous real-valued functions.

2. $C^\infty(M)_{|M^{reg}} \subset C^\infty(M^{reg})$.

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- Assume that L is a compact manifold, $B \subset \mathbb{R}^k$ and $C^\infty(cL(1))$ is a smooth structure on $cL(1)$. $X := B \times cL(1)$.
- A product smooth structure $C^\infty(X)$ is the germ-defined C^∞-ring whose sheaf $SC^\infty(X)$ is generated by $\pi_1^*(SC^\infty(B))$ and $\pi_2^*(SC^\infty(cL(1)))$.

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Smooth structures on stratified spaces

Definition

Assume that $depth(L) = k - 1$, $C^\infty(L)$ be a smooth structure. Let $X := cL(1)$. Let $C^\infty(X^{reg})$ be the product smooth structure on X^{reg}. Then a smooth structure on X is a g.d. C^∞-ring with

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Examples of smooth structures

- $i : X \to \mathbb{R}^n$ s.t. $i(X)$ is locally closed. $C^\infty(X) := i^*(C^\infty(\mathbb{R}^n))$.
- M/G, where $G \subset Diff(M)$ is compact. $C^\infty(M/G) := C^\infty(M)^G$.
- Resolvable smooth structure Assume that we have a continuous projection $M \xrightarrow{\pi} X$ from a smooth manifold M with corner to a stratified space X such that for each stratum $S_i \subset X$ the triple $(\pi^{-1}(S_i), \pi_i, S_i)$ is a differentiable fibration. Then $C^\infty(X) := \{f \in C^0(N) | \pi^* f \in C^\infty(M)\}$ is a smooth structure.
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Two structure theorems

Lemma

For any locally finite open covering $\{U_i\}$ of M there exists a smooth partition of unity subordinate to U_i (i.e. there are nonnegative smooth functions $f_i \in C^\infty(X)$ with support in U_i satisfying $\sum f_i = 1$).

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Any resolvable smooth structure on X obtained from a smooth manifold M is not **finitely generated**, *if there exists $x \in X$ such that $\dim \pi^{-1}(x) \geq 1$, where $\pi : M \to X$ is the associated projection.*

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Germs of 1-forms

- $C_x^\infty(X)$ is a local $\mathbb{R}$-algebra with the unique maximal ideal $\mathfrak{m}_x$ consisting of functions vanishing at x. Set $T_x^*(X) := \mathfrak{m}_x/\mathfrak{m}_x^2$.
- Using $0 \to \mathfrak{m}_x \to C_x^\infty \xrightarrow{j} \mathbb{R} \to 0$ we define the Kähler derivation $d : C_x^\infty(X) \to T_x^* X$ by: $d(f_x) = (f_x - j^{-1}(f_x(x)) + \mathfrak{m}_x^2$.
- We call $\Omega_x^1(X) := C_x^\infty(X) \otimes_{\mathbb{R}} \mathfrak{m}_x/\mathfrak{m}_x^2$ the germs of 1-forms at x.

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Derivation of a germ of k-forms

Set $\Omega^k_x(X) := C^\infty_x(X) \otimes_{\mathbb{R}} \Lambda^k(\mathfrak{m}_x/\mathfrak{m}_x^2)$. Then $\oplus_k \Omega^k_x(X)$ is an exterior algebra with the following wedge product
$(f \otimes_{\mathbb{R}} dg_1 \wedge \cdots \wedge dg_k) \wedge (f' \otimes_{\mathbb{R}} dg_{k+1} \wedge \cdots \wedge dg_l) =$
$(f \cdot f') \otimes_{\mathbb{R}} dg_1 \wedge \cdots \wedge dg_l,$
where $f, f' \in C^\infty_x$ and $dg_i \in T^*_x M$.
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Set $\Omega^k_x(X) := C^\infty_x(X) \otimes_{\mathbb{R}} \Lambda^k(\mathfrak{m}_x/\mathfrak{m}_x^2)$. Then $\oplus_k \Omega^k_x(X)$ is an exterior algebra with the following wedge product
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Differential k-forms

- A section $\alpha : X \to \Lambda^k T^*(X)$ is called a smooth differential k-form, if $\forall x \in X$ there exists $U(x) \subset X$ such that $\alpha(x) = \sum_{i_0 i_1 \cdots i_k} f_{i_0} df_{i_1} \wedge \cdots df_{i_k}$ for some $f_{i_0}, \cdots, f_{i_k} \in C^\infty(X)$.
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Symplectic stratified spaces

- A stratified space X is called symplectic, if every stratum S_i is provided with a symplectic form ω_i. The collection $\{\omega_i\}$ is called *a stratified symplectic form* or simply *a symplectic form*.
- **Examples**. - $S^2 \times S^2 = D^4 \cup D^2 \cup D^2 \cup \{pt\}$.
  - Any contact 3-manifold (M^3, α) is a concave boundary of some symplectic manifold (M^4, ω). Attaching a symplectic cone $cM^3(1)$

$$\bar{\omega}([z, t]) = \frac{1}{2} t^2 d\alpha(z) + t dt \wedge \alpha(z),$$

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Outline

1. Stratified spaces and their smooth structures
   - Stratified spaces
   - Smooth structures on stratified spaces
   - Structures theorems
2. Compatible smooth structures on symplectic stratified spaces
   - Symplectic stratified spaces
   - Compatible smooth structures
3. Properties of compatible smooth structures on stratified symplectic spaces
   - Brylinski-Poisson homology
   - Existence of Hamiltonian flow
   - Leftschetz decomposition
4. End

Smooth structures on stratified symplectic spaces

Let (X, ω) be a symplectic stratified space and $C^\infty(X)$ be a smooth structure on X.

1. A smooth structure $C^\infty(X)$ is said to be weakly symplectic, if there is a smooth 2-form $\tilde{\omega} \in \Omega^2(X)$ such that the restriction of Ω to each stratum S_i coincides with ω_i. In this case we also say that ω is compatible with $C^\infty(X)$, and $C^\infty(X)$ is compatible with ω.

2. A smooth structure $C^\infty(X)$ is called Poisson, if there is a Poisson structure $\{,\}_\omega$ on $C^\infty(X)$ such that $(\{f, g\}_\omega)_{|S_i} = \{f_{|S_i}, g_{|S_i}\}_{\omega_i}$ for any stratum $S_i \subset X$.

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Example of compatible smooth structures I

We assume that a compact Lie group G acts on a symplectic manifold (M, ω) with proper moment map $J : M \to \mathfrak{g}^*$. Let $Z = J^{-1}(0)$. The quotient space $M_0 = Z/G$ is called a symplectic reduction of M. $C^\infty(M_0)_{can} := C^\infty(M)^G / I^G$, where I^G is the ideal of G-invariant functions vanishing on Z.

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- For $x \in \mathfrak{g}$ let $x = x_s + x_n$ be the Jordan decomposition of x, where $x_n \neq 0$ is a nilpotent element, x_s is a semisimple and $[x_s, x_n] = 0$. Assume that x_n is a minimal nilpotent element in $\mathcal{Z}_{\mathfrak{g}}(x_s)$. Then $\overline{G(x)}$ is a stratified symplectic space of depth 1. The embedding $\overline{G(x)} \to \mathfrak{g}$ provides $\overline{G(x)}$ with a natural finitely generated C^∞-ring

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Outline

Symplectic homology

Assume that $C^\infty(M)$ is Poisson.

- $\to \Omega^{n+1} \xrightarrow{\delta} \Omega^n(M) \to \ldots$
- $\delta(f_0 df_1 \wedge \cdots df_n) := \sum_{i=1}^n (-1)^{i+1} \{f_0, f_i\}_\omega df_1 \wedge \cdots \wedge \widehat{df_i} \wedge \cdots \wedge df_n$ $+ \sum_{1\le i<j\le n} f_0 d\{f_i, f_j\}_\omega \wedge df_1 \wedge \cdots \wedge \widehat{df_i} \wedge \cdots \wedge \widehat{df_j} \wedge \cdots \wedge df_n.$
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Brylinski-Poisson homology of compatible Poisson smooth structures

Theorem

Suppose (X, ω) is a stratified symplectic space provided with a Poisson smooth structure $C^\infty(M)$ which is compatible with symplectic from ω. Then the symplectic homology of the complex $(\Omega(X), \delta)$ is isomorphic to the deRahm cohomology with inverse grading : $H_k(\Omega(X), \delta) = H^{m-k}(\Omega, d)$. It is equal to the singular cohomology $H^{m-k}(X, \mathbb{R})$, if the smooth structure is locally contractible.

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Existence of Hamiltonian flow I

Let (X, ω) be a stratified symplectic space and $C^\infty(X)$ a Poisson smooth structure on X.

Lemma

For any $H \in C^\infty(X)$ the associated Hamiltonian vector field X_H defined on X by setting

$$X_H(f) := \{H, f\}_\omega \text{ for any } f \in C^\infty(X)$$

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Theorem

Given a Hamiltonian function $H \in C^\infty(X)$ and a point $x \in X$ there exists a unique smooth curve $\gamma : (-\varepsilon, \varepsilon) \to X$ such that for any $f \in C^\infty(X)$ we have

$$\frac{d}{dt} f(\gamma(t)) = \{H, f\}.$$

The decomposition of X into strata of equal dimension can be defined by the Poisson algebra of smooth functions.

Outline

Leftschetz decomposition of $\Omega(X)$

Let L be the wedge multiplication by ω.

$$\mathcal{P}_{n-k}(X^{2n}) := \{\alpha \in \Omega^{n-k}(X^{2n}) | L^{k+1}\omega = 0\}.$$

Theorem

We have the following decomposition for $k \geq 0$

$$\Omega^{n-k}(X^{2n}) = \mathcal{P}_{n-k}(X^{2n}) \oplus L(\mathcal{P}_{n-k-2}(X^{2n})) \oplus \cdots$$

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$$\mathcal{P}_{n-k}(X^{2n}) := \{\alpha \in \Omega^{n-k}(X^{2n}) | L^{k+1}\omega = 0\}.$$

Theorem

We have the following decomposition for $k \geq 0$

$$\Omega^{n-k}(X^{2n}) = \mathcal{P}_{n-k}(X^{2n}) \oplus L(\mathcal{P}_{n-k-2}(X^{2n})) \oplus \cdots$$

$$\Omega^{n+k}(X^{2n}) = L^{k}(\mathcal{P}_{n-k}(X^{2n})) \oplus L^{k+1}(\mathcal{P}_{n-k-2}(X^{2n})) \oplus \cdots$$

THANK YOU!

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